A practical gauge invariant regularization of the SO(10) grand unified model.

M.M.Deminov[1] and A.A.Slavnov[1,2].

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[1]*Moscow State University, Moscow, 117234, Russia.*
[2]*Steklov Mathematical Institute, Russian Academy of Sciences,*
Gubkina 8, Moscow, 117966, Russia.

Abstract

It is shown that a simple modification of the dimensional regularization allows to compute in a consistent and gauge invariant way any diagram with less than four loops in the SO(10) unified model. The method applies also to the Standard Model generated by the symmetry breaking $SO(10) \rightarrow SU(3) \times SU(2) \times U(1)$. A gauge invariant regularization for arbitrary diagram is also described.

I. Introduction.

An invariant regularization is an important part of renormalization of gauge invariant models. Although in principle any regularization procedure may be used, symmetry breaking regularization requires non gauge invariant counterterms, which makes renormalization quite cumbersome.

In practical calculations the dimensional regularization (see [1]) was mainly used so far. However, the dimensional regularization is not applicable to the models with chiral fermions as in this method there is no symmetry preserving consistent definition of γ_5-matrix. This means in particular that the dimensional regularization does not preserve the symmetry of the Standard Model and grand unified models which reduce at low energies to the Standard Model. At one loop level it is not a very serious problem, as one can find relatively easy counterterms which restore gauge invariance. However for diagrams including higher loops this procedure becomes more and more complicated.

Another gauge invariant regularization is provided by the higher covariant derivative (HCD) method [2, 3]. This regularization has the advantage of being implementable at the Lagrangian level practically to any model. However it does not provide a complete regularization: one loop diagrams remain divergent. For pure Yang-Mills theory the additional gauge invariant regularization of one loop diagrams was proposed in reference [4]. Some problems related to this additional regularization were discussed in references [5, 6]. The complete and self-consistent procedure of HCD regularization is described in the paper [7] (see also [8]).

However for models with chiral fermions the problem still existed as no invariant regularization for the one loop fermion diagrams was known. For the case of the Standard Model this problem was solved in the paper [9]. The procedure developed in [9] combined

with the HCD method of the paper [7] provides a complete gauge invariant regularization of the Standard Model.

Nevertheless for practical calculations this procedure is not very convinient due to a complicated structure of regularized Lagrangian and a simpler method would be welcome.

Theoretical analysis of the present experiments on searches of Higgs meson and determination of quark mass requires calculation of two loop diagrams, so a construction of a simple symmetry preserving regularization is of great practical importance.

In the present paper we propose a new gauge invariant regularization method for the unified SO(10) model which combines ideas of different approaches [1, 2, 9]. For the one, two and three loop diagrams which at present are the most important from the practical point of view we demonstrate that a simple modification of dimensional regularization is sufficient to provide a gauge invariant calculation procedure.

Our method is also applicable to the Standard Model as it may be obtained from the unified SO(10) model via breaking SO(10) gauge group to SU(3) × SU(2) × U(1). This symmetry breaking may occur spontaneously via Higgs mechanism if one insists on the unification of all interactions at very high energies, or may be introduced explicitly if one is interested only in study of the Standard Model at low and intermediate energies. In this paper we show that such symmetry breaking is compatible with our method. A detailed discussion will be given elsewhere.

For a general multiloop diagram this method is not sufficient (see for example diagrams on figures 4 and 5 in Appendix). In this case the gauge invariant regularization may be achieved by combining the dimensional regularization with the HCD method. It is important to notice that in our case it is sufficient simply to introduce higher covariant derivatives in the Lagrangian and no additional problems with one loop diagrams arise. In this way one can avoid the most complicated part of the HCD method and calculations remain reasonably simple.

II. Modified dimensional regularization for the diagrams with less then four loops.

The difficulties of applying dimensional regularization to chiral models are due to impossibility of a consistent symmetry preserving definition of the γ_5-matrix. The usual definition $\gamma_5 = \frac{i}{4!}\varepsilon^{\mu\nu\rho\sigma}\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma$ involves the totally antisymmetric tensor $\varepsilon^{\mu\nu\rho\sigma}$, which has no natural extension beyond $d = 4$. One can try to define γ_5 in arbitrary dimension axiomatically, postulating that it anticommutes with all γ_μ-matrices (see e.g. [10]). It was shown however that such a definition contradicts the cyclicity property of γ-matrices trace (see e.g. [11]).

Another definition was proposed by G.'tHooft and M.Veltman [1] who postulated that $\bar{\gamma}_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$ in arbitrary dimension. This definition is obviously self-consistent, but due to the fact that $\bar{\gamma}_5$ anticommutes with γ_μ, $\mu = 0, 1, 2, 3$, and commutes with all other matrices, it breaks chiral invariance and hence the gauge invariance. Of course, the gauge invariance of renormalized theory may be restored by making subtractions in such a way to provide Slavnov-Taylor identities [12, 13] for renormalized Green functions, but it complicates the job drastically. The detailed analysis of this approach was carried out by P.Breitenlohner and D.Maison. [14]. The symmetry breaking in this approach arises from two sources. Any convergent diagram Π_c may acquire a noninvariant piece, vanishing at $d = 4$:

$$\Pi_c = \Pi_c^{inv} + (d - 4)\tilde{\Pi}. \tag{1}$$

The term $\tilde{\Pi}$ by itself is harmless as long as the diagram Π is not a subgraph of some divergent diagram. In the later case Π_c is multiplied by the pole term proportional to

$1/(d-4)$ and $\tilde{\Pi}$ gives a nonzero symmetry breaking contribution.

For divergent diagram Π_d the limit $d \to 4$ does not exist and the diagram by itself may acquire a finite or infinite noninvariant term

$$\Pi_d = \Pi_p^{inv} + \frac{1}{d-4}\Pi' + \Pi'' \tag{2}$$

We stress that with this prescription breaking of gauge invariance may occur not only in fermion loop, but also in diagrams involving open fermion lines.

On the contrary, the prescription of total anticommutativity meets no problems in diagrams without fermion loops and preserves gauge invariance. In this case the problem is related solely to fermion loops.

Below we shall show that for any diagram with less then four loops in the SO(10) unified model as well as for a large class of higher loop diagrams this problem does not arise as these diagrams in fact do not depend on γ_5 matrix properties in dimensions $d \neq 4$. The proof will essentially follow the ideas of the paper [9].

The gauge invariant SO(10) Lagrangian with spinor fields is chosen in the form:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + i\overline{\psi}\gamma_\mu(\partial_\mu - igA_\mu^{ij}\sigma_{ij})\psi. \tag{3}$$

Here the spinors ψ span the 16 dimensional representation of SO(10). The spinor fields have positive chirality with respect to the Lorentz group: $\psi = \frac{1}{2}(1+\gamma_5)\psi$. The spinors ψ include left components of quarks and leptons and their charge conjugated right components. The fifteen components of ψ describe exactly one generation of the Standard Model. The remaining component corresponds to the right-handed neutrino. It is a singlet with respect to SU(3) × SU(2) × U(1) group, and if one is interested only in the description of the Standard Model, this component may be omitted.

The 16-dimensional irreducible representation of SO(10) is obtained by applying the projector to 32-dimensional vector: $\psi = \frac{1}{2}(1+\Gamma_{11})\psi$. The matrices σ_{ij} are the SO(10) generators: $\sigma_{ij} = \frac{i}{2}[\Gamma_i, \Gamma_j]$, where Γ_i are Hermitian 32 by 32 matrices which satisfy the Clifford algebra: $\{\Gamma_i, \Gamma_j\} = 2\delta_{ij}$; $\Gamma_{11} = -i\Gamma_1\Gamma_2\cdots\Gamma_{10}$. Structure of these 32 by 32 matrices is presented in [15].

The SU(3) × SU(2) × U(1) invariant Lagrangian may be obtained either by putting all gauge fields except for the ones corresponding to the generators of this subgroup equal to zero, or by introducing proper Higgs fields. Further reduction of the symmetry group to SU(3) × U(1) also may be provided by condensate of Higgs fields. We postpone a detailed discussion of this procedure till special publication, but demonstrate now that introduction of Higgs fields interaction may be easily incorporated in our method. In particular the masses for W and Z mesons are generated by the following Higgs field Lagrangian:

$$\mathcal{L}_H = \frac{1}{32}\mathrm{tr}(D_\mu\phi)^\dagger D_\mu\phi - \frac{1}{2}\psi^{\mathrm{T}}C_D C(\phi+\phi^\dagger)\psi + \frac{1}{2}\bar{\psi}C_D(\phi+\phi^\dagger)C\bar{\psi}^{\mathrm{T}} - \lambda(\mathrm{tr}(\phi^\dagger\phi)-\mu^2)^2. \tag{4}$$

Here the Higgs field ϕ is 32 × 32 matrix: $\phi = \phi_i\Gamma_i$, ϕ_i are complex, $i = 1\ldots10$ (here and below summation over repeated indices is assumed), $D_\mu\phi = \partial_\mu\phi - ig[A_\mu^{ij}\sigma_{ij}, \phi]$. The matrix C_D is a charge conjugation matrix. The matrix C is a conjugation matrix defined by the relation $\sigma_{ij}^{\mathrm{T}}C = -C\sigma_{ij}$, which provides gauge invariance of second and third terms in (4). Matrix C anticommutes with Γ_{11} : $C\Gamma_{11} = -\Gamma_{11}C$.

Spontaneous symmetry breaking is due to nonzero vacuum expectation value of the Higgs field $\langle\phi\rangle$:

$$\langle\phi\rangle = \frac{\mu}{4\sqrt{2}}(\alpha\Gamma_3 + \beta\Gamma_4), \quad \alpha\alpha^\dagger + \beta\beta^\dagger = 1. \tag{5}$$

The usual perturbation theory arises after the shift of Higgs fields by the stationary value (5). However for a general analysis it is more convinient to work in terms of unshifted fields. Then the mass terms arise due to interaction with condensate: summation of the series of insertions of arbitrary number of scalar vertices describing the interaction of fermion, scalar or vector fields with the condensate results in the shift of the masses in the corresponding propagators.

The useful observation is that the theory described by the Lagrangian (3), (4) is invariant under simultaneous change of signs of γ_5 and Γ_{11} matrices. Indeed, these Lagrangians may be rewritten in terms of transposed fields:

$$\mathcal{L}_\psi = -i\partial_\mu\psi^{\mathrm{T}}\frac{(1+\gamma_5^{\mathrm{T}})}{2}\frac{(1+\Gamma_{11}^{\mathrm{T}})}{2}\gamma_\mu^{\mathrm{T}}\bar\psi^{\mathrm{T}} - g\psi^{\mathrm{T}}\frac{(1+\gamma_5^{\mathrm{T}})}{2}\frac{(1+\Gamma_{11}^{\mathrm{T}})}{2}\sigma_{ij}^{\mathrm{T}}A_\mu^{ij}\gamma_\mu^{\mathrm{T}}\bar\psi^{\mathrm{T}}. \tag{6}$$

Let us multiply ψ^{T} by the unit factor CC_DCC_D. Commuting CC_D to the right we get

$$\mathcal{L}_\psi = i\psi^{\mathrm{T}}CC_D\frac{(1+\gamma_5)}{2}\frac{(1-\Gamma_{11})}{2}(-\gamma_\mu)CC_D\partial_\mu\bar\psi^{\mathrm{T}}-$$

$$- g\psi^{\mathrm{T}}CC_D\frac{(1+\gamma_5)}{2}\frac{(1-\Gamma_{11})}{2}(-\sigma_{ij})A_\mu^{ij}(-\gamma_\mu)CC_D\bar\psi^{\mathrm{T}}. \tag{7}$$

Introducing the conjugated fields $\psi^c = CC_D\bar\psi^{\mathrm{T}}$, one can write this equation in the form:

$$\mathcal{L}_\psi = -i\bar\psi^c\gamma_\mu(\partial_\mu - igA_\mu^{ij}\sigma_{ij})\frac{(1-\gamma_5)}{2}\frac{(1-\Gamma_{11})}{2}\psi^c. \tag{8}$$

Similarly, the Higgs-fermion interaction Lagrangian is rewritten as

$$\mathcal{L}_{\phi\psi} = -\frac{1}{2}\psi^{c\mathrm{T}}C_DC(\phi+\phi^\dagger)\frac{(1-\gamma_5)}{2}\frac{(1-\Gamma_{11})}{2}\psi^c + \frac{1}{2}\bar\psi^cC_D(\phi+\phi^\dagger)C\frac{(1+\gamma_5)}{2}\frac{(1-\Gamma_{11})}{2}\bar\psi^{c\mathrm{T}}, \tag{9}$$

It follows from equations (8),(9) that simultaneous change of the signs of γ_5 and Γ_{11} does not influence the value of any diagram. In particular, if a diagram does not involve Γ_{11}, it may be written in the form which also does not involve γ_5-matrix. This property is very important for us, and it will be used below.

Now let us consider arbitrary one loop fermion diagram in the model described by Lagrangian (3),(4). Any such loop is proportional to

$$\mathrm{tr}\left[\frac{(1+\Gamma_{11})}{2}\sigma^{i_1i_2}\cdots\Gamma^{i_k}\cdots\sigma^{i_{n-1}i_n}\right] \tag{10}$$

– a trace of a product of the projector $\frac{1}{2}(1+\Gamma_{11})$ and σ- and Γ-matrices, which correspond to external vector and scalar lines respectively. Each σ matrix is a product of two different Γ matrices. Hence the trace of the term, proportional to Γ_{11} is zero if the number n is less than 10, in other words the Γ_{11} under the trace is multiplied by less then 10 Γ-matrices. That means that all one loop fermion diagrams for which the sum of the number of vector external lines and one half of the number of scalar external lines is less then 5 do not involve Γ_{11} and therefore do not depend on γ_5.

Moreover one can easily see that these diagrams up to the factor $1/2$ coincide with the corresponding diagrams in the model where the fermions span the 32-dimensional representation of SO(10). This model is vectorlike and may be rewritten in a purely vectorial form.

It is proven in the Appendix that this property (absence of γ_5) remains valid for a fermion loop with arbitrary boson insertions if the sum of the number of vector external lines and one half of the number of scalar external lines is less then 5.

That means one can apply to such diagrams dimensional regularization with the prescription of total anticommutativity of γ_5 matrix. The procedure is manifestly gauge invariant, and no problems with the γ-matrices traces arise.

A slight modification of this discussion allows to extend our procedure to the diagrams with arbitrary number of external vector or scalar lines. All diagrams with one fermion loop for which the sum of the number of vector external lines and one half of the number of scalar external lines is more then 5, that is the diagrams with 10 or more SO(10)-group indices corresponding to external vector or scalar lines are superficially convergent. To renormalize such a diagram it is sufficient according to R-operation to subtract the counterterms corresponding to all divergent subgraphs. As was discussed above these subgraphs may be calculated using the dimensional regularization and minimal subtractions. After such subtraction the corresponding integral becomes convergent and one can remove the dimensional regularization before calculating the trace over the fermion loop, where γ_5 can give nonzero contribution. The calculation of this trace has to be carried out in four-dimensional space-time, hence there are no problems with γ_5.

This remark is particularly important for taking into account the Higgs field condensate contribution. We remind that in our discussion we assume that the fermion masses in propagators arise via summation of insertions of Higgs field condensate to fermion lines. That means some "external" Higgs field line are fictitious and amount only to the shift of fermion masses. These insertions are proportional to $\alpha\Gamma_3 + \beta\Gamma_4$ and several insertions may generate additional factors σ_{34} in the trace (10). Presence of such factors do not change our reasoning for diagrams with two or three "real" external lines. To make the traces in these diagrams different from zero, one needs at least two different σ-matrices. The mass insertion cannot provide two different σ-matrices.

The situation is different for the diagrams with four "real" external lines. In this case additional matrix σ_{ab} arising from mass insertions can make the trace with factor γ_5 different from zero. However such a diagram is superficially convergent and as was discussed above also allows the dimensional regularization. Obviously analogous arguments may be applied if one works in terms of "shifted" Lagrangian generating massive propagators.

The same arguments are applicable to other Higgs field interaction: insertion of Higgs field condensates to fermion lines either do not introduce γ_5 matrix, or makes the diagram finite.

Another class of diagrams which allow a straight-forward dimensional regularization with the γ_5 matrix defined via anticommutativity with all γ_μ consists of the diagrams without fermion loops. In this case there is no need to calculate the trace over spinoral indices and the gauge invariance is preserved.

So we showed that a large class of the diagrams in the unified SO(10) model may be computed using the dimensional regularization with additional prescriptions formulated above.

The diagrams which do not fall into this class are presented by the superficially divergent diagrams where open fermion line or fermion loop for which the sum of number

of vector lines and the one half of the number of scalar lines is more then 5 is linked to a fermion loop for which the sum of number of vector lines and the half of the number of scalar lines is more then 5. (see figures 4 and 5 respectively). The lowest order diagrams of this type include 4 loops.

To deal in a gauge invariant way with these diagrams one can use a hybrid regularization which combines the dimensional regularization and HCD method.

III. The Hybrid gauge invariant regularization.

There are several possibilities for combining the dimensional regularization with HCD method. A particular choice may be done on the basis of the most simple calculations of the diagram in question. As at the moment calculations of four and higher loop diagrams in the Standard Model are of mainly academic interest we present here only one, conceptually the most straight-forward method.

The regularized Lagrangian may be chosen in the form:

$$\mathcal{L}_\Lambda = -\frac{1}{4}(F_{\mu\nu}^2 + \frac{1}{\Lambda^2}D_\alpha F_{\mu\nu}D_\alpha F_{\mu\nu}) + i\bar{\psi}\gamma_\mu D_\mu\psi + \frac{1}{2}\mathrm{tr}(D_\mu\phi)^\dagger(D_\mu\phi) +$$

$$+\frac{1}{\Lambda^2}(D^2\phi)^\dagger(D^2\phi) - \frac{1}{2}\psi^\mathrm{T}C_D C(\phi + \phi^\dagger)\psi + \frac{1}{2}\bar{\psi}C_D(\phi + \phi^\dagger)C\bar{\psi}^\mathrm{T} - \lambda(\mathrm{tr}(\phi^\dagger\phi) - \mu^2)^2. \quad (11)$$

In the model described by the Lagrangian (11) all multiloop diagrams are superficially convergent. One loop diagrams with external fermion lines are convergent too. So the only diagrams which need additional regularization are the one loop diagrams without external fermion lines, in particular fermion loops with less then five external fields. As was discussed above these diagrams do not depend on γ_5 and may be described by the effective vectorlike model. Calculation of an arbitrary diagram proceeds as follows. Keeping Λ finite one subtracts the counterterms corresponding to divergent subgraphs using dimensional regularization and the minimal subtractions. As these subdiagrams do not involve γ_5 no problem arises. After that the diagram becomes finite at the dimension $d = 4$. (We remind that the Λ is still finite.)

The second step is removing the higher derivative regularization by taking the limit $\Lambda \to \infty$. This procedure is also manifestly gauge invariant and no symmetry breaking counterterms are needed.

Introduction of higher derivatives makes the calculations in this method more complicated. However using the dimensional regularization allows to avoid the additional regularization of one loop diagrams which is the most cumbersome part of the HCD method.

IV. Discussion.

The procedure described in the section II provides a simple practical method of gauge invariant computation of all diagrams with less then four loops in the unified SO(10) model. This method also applies to the Standard Model which may be obtained via spontaneous symmetry breaking SO(10) $\to$ SU(3) $\times$ SU(2) $\times$ U(1) $\to$ SU(3) $\times$ U(1). Whether it is possible to modify further dimensional regularization to make it applicable to an arbitrary diagram is at present the open question. This question is under investigation. Meanwhile arbitrary diagrams may be treated in a gauge invariant way by means of the hybrid regularization described in section III. We conclude by noticing that the gauge invariance of our procedure was checked at the one loop level by explicit calculation of gluon-W scattering in the Standard Model [16].

Appendix

Theorem : If a diagram contains one fermion loop and no open fermion lines (see figure 1) then matrix Γ_{11} gives a nonzero contribution only if the sum of the number of external vector lines and one half of the number of external scalar lines is equal to 5 or more.
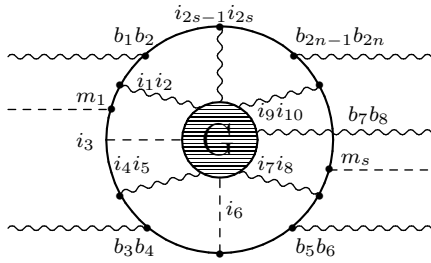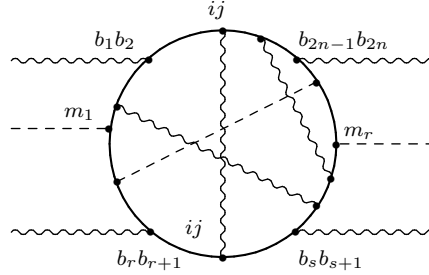

Figure 1.　　　　　　　　　　Figure 2.

Here and in the following dash lines correspond to Higgs particles.

The proof is based on the following identity:

$$\sum_i \Gamma_i \Gamma_{b_1} \cdots \Gamma_{b_t} \Gamma_i = 2 \sum_{l=1}^{t} (-1)^{(l+1)} \Gamma_{b_1} \cdots \Gamma_{b_{l-1}} \Gamma_{b_{l+1}} \cdots \Gamma_{b_t} \Gamma_{b_l} + 10(-1)^l \Gamma_{b_1} \cdots \Gamma_{b_t}. \qquad (12)$$

It is important to note that the number of Γ's in each term of the r.h.s. is equal to the number of Γ's in the original product $\Gamma_{b_1} \cdots \Gamma_{b_t}$ and the indices of Γ's belong to the same set.

Obviously an analogous representation holds if we multiply a product of $\sigma_{b_1 b_2} .. \sigma_{b_{2t-1} b_{2t}}$ by Γ_i from both sides and sum over i. Indeed,

$$\Gamma_{b_i} \sigma_{b_i b_j} \Gamma_{b_i} = \sigma_{b_j b_i}, \quad \text{and} \quad \Gamma_{b_k} \sigma_{b_i b_j} \Gamma_{b_k} = \sigma_{b_i b_j} \text{ if } b_k \neq b_i, b_j. \qquad (13)$$

The product

$$\sigma_{b_1 b_2} \cdots \Gamma_{m_1} \cdots \sigma_{b_l b_{l+1}} \cdots \Gamma_{m_s} \cdots \sigma_{b_{2t-1} b_{2t}} \qquad (14)$$

after multiplication by Γ_i from both sides and summation over i may be represented as a sum of products $\sigma_{\beta_1 \beta_2} \cdots \Gamma_{\mu_1} \cdots \Gamma_{\mu_s} \cdots \sigma_{\beta_{2t-1} \beta_{2t}}$, where the number of σ's and Γ's is the same as in equation (14) and the indices $\beta_1 .. \mu_1 .. \mu_s .. \beta_{2t}$ form some permutation of the indices $b_1 .. b_{2t}, m_1 .. m_s$.

A similar representation holds for the sums

$$\sum_{ij} \sigma_{ij} \sigma_{b_1 b_2} \cdots \Gamma_{m_1} \cdots \sigma_{b_l b_{l+1}} \cdots \Gamma_{m_s} \cdots \sigma_{b_{2t-1} b_{2t}} \sigma_{ij}, \qquad (15)$$

$$\sum_{i} \sigma_{b_0 i} \sigma_{b_1 b_2} \cdots \Gamma_{m_1} \cdots \sigma_{b_l b_{l+1}} \cdots \Gamma_{m_s} \cdots \sigma_{b_{2t-1} b_{2t}} \sigma_{i b_{2t+1}}. \qquad (16)$$

Indeed any σ_{ij} is a commutator of two Γ matrices and applying successively the equation (12) one gets equations (15,16). These equations are sufficient to prove our statement for arbitrary fermion loop, where all the vertices are placed on the loop, and there are no vertices inside the loop (see figure 2).

Indeed, contraction of Yang-Mills or Higgs fields produces the terms proportional to the sums

$$\sum_{ij} \sigma_{ij} \sigma_{b_1 b_2} \cdots \Gamma_{m_1} \cdots \Gamma_{m_s} \cdots \sigma_{b_{2t-1} b_{2t}} \sigma_{ij}.$$

According to discussion presented above after summation over "dummy" indices one gets the sum of products

$$\sigma_{\beta_1 \beta_2} \cdots \Gamma_{\mu_1} \cdots \Gamma_{\mu_s} \cdots \sigma_{\beta_{2t-1} \beta_{2t}},$$

where $\beta_1..\beta_{2t}$, $\mu_1..\mu_s$ are some permutation of external fields indices $b_1..b_{2t+1}$, $m_1..m_s$.

So finally we have to take the traces of the type

$$\mathrm{tr}\Big(\frac{1 + \Gamma_{11}}{2} \sigma_{\beta_1 \beta_2} \cdots \Gamma_{\mu_1} \cdots \Gamma_{\mu_s} \cdots \sigma_{\beta_{2t-1} \beta_{2t}}\Big), \tag{17}$$

where the indices $\beta_1..\beta_{2t}, \mu_1..\mu_s$ form some permutation of external fields indices. By the same arguments which have been given above for one loop diagrams, the trace involving Γ_{11} is zero if the number of external vector fields plus one half of the number of external Higgs fields is less then 5.

Now we extend our proof to fermion loops with arbitrary boson subdiagram. These diagrams may include internal vertices describing the selfinteraction of Yang-Mills fields, selfinteraction of Higgs fields and their mutual interactions.

The threelinear Yang-Mills interaction looks as follows:

$$V_{AAA} = i g t^{(ij)(kl)(mn)}[(p - k)_\rho g_{\mu\nu} + (k - q)_\mu g_{\nu\rho} + (q - p)_\nu g_{\mu\rho}],$$

and the structure constants $t^{(ij)(kl)(mn)}$ are linear combinations of δ's:

$$[\sigma_{ij}, \sigma_{kl}] = t^{(mn)(ij)(kl)} \sigma_{mn},$$

$$t^{(mn)(ij)(kl)} = i\{(im)(ln)(jk) - (km)(jn)(il) - (jm)(nl)(ik) + (km)(in)(lj) -$$
$$- (in)(lm)(jk) + (km)(jm)(il) + (jn)(ml)(ik) - (kn)(im)(lj)\},$$

where $(ij) = \delta_{ij}$.

The fourlinear interaction has a form:

$$V_{AAAA} = g^2\{t^{(ab)(cd)(ij)} t^{(ij)(ef)(hk)}(g_{\mu\rho} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\rho}) + permutations\}.$$

Again the SO(10) structure constants $t^{(ab)(cd)(ij)} t^{(ij)(ef)(hk)}$ consist of δ's: $\delta_{bc}\delta_{de}\delta_{fh}\delta_{ka} + \ldots$

The vertices including the Higgs fields possess analogous property – their SO(10) structure is given by a linear combinations of δ-functions:

$$V_{AA\phi\phi} = g^2\{\delta^{be}\delta^{df}\delta^{ac} - \delta^{ae}\delta^{df}\delta^{bc} - \delta^{be}\delta^{cf}\delta^{ad} + \delta^{ae}\delta^{cf}\delta^{bd}\},$$

$$V_{\phi\phi\phi\phi} = -8\lambda\{\delta^{ef}\delta^{hj} + \delta^{eh}\delta^{fj} + \delta^{ej}\delta^{fh}\},$$

$$V_{A\phi\phi} = ig\{k_\mu \delta^{ea}\delta^{fb} - p_\mu \delta^{eb}\delta^{fa}\},$$

where A's carry SO(10)-indices a, b and c, d; ϕ's have indices e, f, h and j.

Therefore considering an arbitrary diagram without internal spinor lines we may firstly perform the summation over all "dummy" indices. After this summation we shall get the sum of the products of the type (17), where as above the indices $\beta_1..\beta_{2t}, \mu_1..\mu_s$ form some permutation of external lines indices. Hence we arrive to the same conclusion: only diagrams with the number of external vector lines plus one half of the external Higgs lines less the 5 may depend on Γ_{11}.

The theorem does not hold for the diagrams with one fermion loop and open fermion lines (see for example figures 3 and 4).
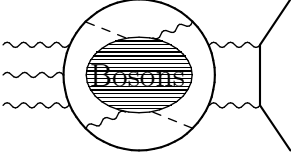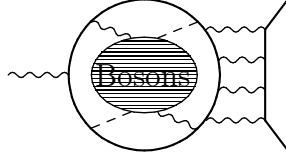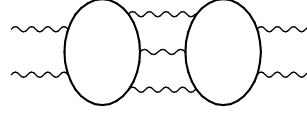

Figure 3.   Figure 4.   Figure 5.

However the diagrams shown at figure 3 are superficially convergent and do not include divergent fermion loops. Hence they may be treated as was explained above. One firstly applies dimensional regularization to calculate the counterterms corresponding to divergent subgraphs, then makes a subtraction according to R-operation. After that the diagram becomes convergent and allows continuation to $d = 4$. The calculation of the trace over the fermion loop may be done in four dimensions where no problems arise.

The only diagrams of this type which cannot be treated in this way are the diagrams with one external vector line and two external fermion lines including a fermion loop with more then four bosonic lines (see for example figure 4). These diagrams are superficially divergent and one cannot remove dimensional regularization before calculating all the traces. The lowest order diagram of this type contains four loops.

The diagrams including more then one fermion loop, like the diagram shown at figure 5 also cannot be treated with the dimensional regularization described above. For their gauge invariant analysis one has to apply the hybrid regularization described in the section III. The lowest order diagram of this type also contains four loops.

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